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Econometric Modeling of the Bank Stress Index-Macroeconomic Fundamental Relationship in Tunisia

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Abstract: This work examines the link between bank stress and a set of macroeconomic variables with regard to the Tunisian context. Indeed, the Tunisian banks' behavior and future strategy cannot remain neutral; especially face to the serious difficulties the Tunisian economy is currently experiencing. The empirical validation of this study involves a sample of ten Tunisian banks' half-yearly data, collected from the Central Bank, covering the period 1998-2021. This econometric work aims to provide significantly interesting results in regard to the Tunisian banking system's vulnerability to certain macroeconomic variables associated with positive shocks, mainly inflation, exports, GDP, *among others*. In addition, the investigated variables are non-stationary in level but stationary in first difference, and the methodology of Johansen-Juselius (1990) makes it possible to empirically demonstrate the persistence of long-term relationships linking such variables as EXP, GDP, INF, TCH, SBP and stress index. At this level, we maintain a long-term relationship indicating that the banking stress index tends to be very elastic with respect to inflation and the exchange rate. It is worth noting, above all, that the correction model estimate of the banking stress index in Tunisia proved to demonstrate a short-term variation, absolutely, insensitive to the adjustments of the various variables investigated, particularly; exports, GDP, inflation, exchange rate and balance deficit.

Keywords: Stress Index, Banks, Macroeconomic Variables, Fragility

JEL Classification Number: G21, E, F62, G01

1. Introduction

The present research is focused on identifying and measuring the macroeconomic elements likely to maintain the Tunisian banking system's financial efficiency. To this end, we consider investigating the banking stress and a number of macroeconomic variables binding relationship. Actually, the simulation findings turn out to reveal that the banking supervisory authorities are required to probe the impact of macroeconomic blows or shocks, highly affecting and striking the Tunisian banking activity.

In this respect, a wide range of empirical research works dealing with this subject in several contexts are found in the relevant literature, worth citing among them are, mainly,

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Apergis et al. (2019), Dua and Kapur (2018), Kanas and Molyneux (2018), Monnin (2005), aw wall as Demirgûç-Kunt and Detragiache (1998)), to cite but a few. Noteworthy, however, is that most of the already conducted works appear to apply either the error correction modeling, or the estimation of impulse functions (IRF functions) econometric methods on treating this issue.

With regard to the present work, however, we consider assessing the Tunisian banking system's stress resistance level or capacity face to macroeconomic shocks (IRF function from a VECM or VAR), as well as a shock on exogenous variables (error-correction along with ECM modeling).

It is worth highlighting, in this respect, that the ECM modeling strategy is applied in our context to help study the relationship between the Tunisian stress index and such macroeconomic variables as: the GDP growth rate (GDP), current balance as a percentage of GDP (SCP), budgetary balance as a percentage of GDP (SBP), inflation rate (INF), exchange rate (TCH), export growth rate (EXP), and external debt (DTex). Such a procedure is undertaken for the purpose of empirically analyzing the interaction between a selection of macroeconomic variables and the Tunisian banking stress index on a biannual basis, regarding the time period: 1998-2021. Indeed, this econometric framework is intended to provide us with highly interesting signs and indications regarding the Tunisian banking system's vulnerability extent following a positive shock effect on such macroeconomic variables as inflation, exports, GDP, etc. We then proceed with exploring the banking stress dynamics following a macroeconomic shock, whereby, analysis of the interaction between the bank stress index and a number of macroeconomic variables is theoretically and empirically approached. At this level, the Granger causality test proved to demonstrate a two-way causation with respect to most of administered tests (Table 6).

Accordingly, response to our already posited research question entails, among other techniques, the implementation of the VECM or VAR models, to be able to estimate the impulse response functions, fit for analyzing the bank stress-index responses to a random shock on the so-dubbed 'impulse variable' with a macroeconomic aspect. Basically, these stimulus functions are liable to stimulate some of the macroeconomic variables relevant responses, as well as those associated with the banking stress index, following structural macroeconomic shocks. For instance, the IRF functions would enable us to trace, over a ten-period horizon, both of the macroeconomic variables and the bank stress index associated dynamics following an orthogonal shock. It is important to note that this behavior determining method (of banking as well as economic fundamentals stress level) regarding a positive shock on the horizon has been applied in several conducted research papers (e.g., d'Apergis et al., 2019). Other empirical works, however (e.g., Dua and Kapur, 2018), tend to implement error-correcting econometric models to determine the macroeconomic variables significantly influencing the banking stress dynamics and, there from, the banking system's fragility. Such an approach entails using the Johansen-Juselius method (1990) to retrieve any possible long-term relationships binding the entirety of the model associated variables, which need be integrated in order of one. To this end, such stationarity tests as the ADF and KPSS have been administered.

It is also worth specifying that on estimating the impulse response functions of some relevant fundamentals (exports, GDP, inflation rate, budgetary balance, external debt, and current balance) and of the banking stress variable, the latter must figure ultimately within the causal chain. By means of example, to determine the reaction of the bank stress index throughout an exchange rate (CH) depreciation and a public debt (DP) increase span, the following causal chain must be considered: CH→DP→Bank Stress Index. Such a chain would help explore the exchange rate as well as the public spending associated dynamics following a positive shock on the banking stress index. Prior to elaborating the empirical work, we proceed with summing up the Johansen's procedure, and the ensuing ECM models, which also served us to draw and estimate the impulse functions out of a structural VAR model.

Overall, our study turns out to involve three main parts. The first section is devoted to highlighting the applied variables and methodology applied. As to the second section; it is devoted to depicting the empirical validation framework, while the achieved empirical results and relevant interpretations make up the subject of the ultimate section.

2. Econometric Methodology

As already stated, the present study is conducted to address the issue of macroeconomic variables' effect on banking stress index via ECM modeling, through the prerequisite of administering a cointegration test. In this respect, the most powerful and widely applied test turns out to be the Johansen-Juselius (1990) test. Prior to administering our empirical tests, a summary presentation of our applied techniques seems worth outlining, for simplicity purposes of the nature ofour applied econometric models and a thorough understanding of our reached empirical results.

2.1. The Johansen-Juselius methodology and ECM modeling

The Johansen's procedure is aimed to test whether a set of N economic and/or financial variables would demonstrate or preserve the same trend over the very long run. This technique serves to check whether two variables, for instance, would maintain the same dynamics over a very long term so that the difference between them remains stationary. This logic could also be generalized to involve a set of N variables, wherein, their binding cointegration would denote the persistence of a common tendency among them, i.e., the gap between these variables tends to remain noticeably stable over the time period subject of study. In respect of our examined data, the below figuring diagram (Figure 1) empirically illustrates the persistence of a common tendency in the evolution of the following macroeconomic variables: EXP, IMP, INF (i.e., the growth rates of exports, imports and inflation, respectively)

The relevant table (table 2) highlights well that these three stated macroeconomic variables prove to display a common trend, there from, a stationary gap throughout the study period. One could well conclude, therefore, that these variables are liable to display a long-term relationship. Aplausible confirmation of this graphic analysis (figure1) is provided by the method of Johanson, which supposes a vector Y_t of N random variables that follows an autoregressive vector process of order p, i.e., a VAR (p), formally expressed by Johansen-Juselius (1988-1990) as:

$$Y_t = \prod_1 Y_{t-1} + \prod_2 Y_{t-2} + \prod_3 Y_{t-3} + \ \dots\dots\dots + \prod_p Y_{t-p} + \varepsilon_t \qquad (1)$$

Johansen-Juselius (1988-1990) assume that the vector innovations $\varepsilon_t$follow the multidimensional normal law: $\varepsilon_t \rightsquigarrow \mathcal{N}\left(\underline{0}, \Gamma\right),$ and that the entirety of the components of $Y_t$are integrated variables of order 1, i.e., the first transformation of the components of Y_t is stationary. Accordingly, the transition to a stationary VAR requires the parameterization of equation (1) as follows:

$$\Delta Y_t = \breve{\Pi}_1 \Delta Y_{t-1} + \breve{\Pi}_2 \Delta Y_{t-2} + \breve{\Pi}_3 \Delta Y_{t-3} + \cdots + \breve{\Pi}_{p-1} \Delta Y_{t-p+1} + \breve{\Pi}_p Y_{t-p} + \varepsilon_t \qquad (2)$$

$$\text{or, } \breve{\Pi}_i = -\mathrm{I} + \prod_1 + \prod_2 + \cdots \dots \dots .. + \prod_i \ / \ i = 1 \rightarrow p \qquad (3)$$

Equation (2) pinpoints, a priori, a persistent a 'disorder' at the level of the integration order among the equation variables, particularly noticeable in the quantity $\breve{\Pi}_p Y_{t-p}$. Worth recalling, however, is that the by-product of the matrix $\breve{\Pi}_p$ and vector Y_{t-p}could yield a stable vector provided that there exists at least a long-term relationship among the various components of Y_{t-p}. Such a relationship results in maintaining a balance in the integration order between the above stated VAR various components. The number of long-term relationships could by no means not exceed the value N-1. This has its explanation in the fact that the matrix $\breve{\Pi}$ is non-invertible, thereby, its associated rank should reflect the number of long-term relationships, which is strictly inferior to N[1].

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[1]The non-inversion of the matrix $\breve{\Pi}$ can be demonstrated by the absurd. We set $\breve{\Pi}_p Y_{t-p} = v_t$, since the components of the vector v_t are stationary as well as the product $\Pi_p^{-1} v_t$ is equal to Y_{t-p}. This equality is not possible since the components of Y_{t-p} are non-stationary and integrated of order 1 by assumption in the procedure of Johansen (the components of Y_{t-p} are integrated of order 1. Thus, such an equality is dummy since on the one hand we have a stationary variable and on the other hand another non-stationary one. So the matrix $\breve{\Pi}_p$ is not invertible and thus its rank is completely lower than N noted r in the procedure of Johansen and which expresses the number of cointegration vectors.

It is also important to note that the matrix $\breve{\Pi}$ is expressed in the form of the product of two matrices, wherein, JJ denote α and β, respectively. According to Engle and Granger (1987), these matrices stand as matrices of adjustment coefficients and cointegration coefficients of rank (N, r), respectively. On substituting $\breve{\Pi}_p$ by the equivalent Engle and Granger demonstrated expression in equation (2), the following expression is reached:

$$\Delta Y_t = \breve{\Pi}_1 \Delta Y_{t-1} + \breve{\Pi}_2 \Delta Y_{t-2} + \breve{\Pi}_3 \Delta Y_{t-3} + \cdots \ldots . + \breve{\Pi}_{p-1} \Delta Y_{t-p+1} + \alpha \beta' Y_{t-1} + \varepsilon_t \quad (4)$$

Determination of the value of r is then implemented through administration of two widely recognized tests: the Trace test as well as the maximum eigenvalue test.

2.1.1. Trace test

It is a sequential test enabling to gradually determine the number of cointegration vectors among the N variables in question. In this respect, Johansen-Juselius (1990) put forward the following hypotheses:

$$\begin{cases} H_0: rg(\breve{\Pi}_p) \leq r \\ H_1: rg(\breve{\Pi}_p) \geq r+1 \end{cases} \quad (5)$$

To test the basic hypothesis, Johansen and Juselius considered applying the LR likelihood ratio statistic, defined as:

$LR = -T \sum_{i=r+1}^{N} \log\left(1 - \hat{\lambda}_i\right) \rightsquigarrow$ Non-Standard Law, expressed in terms of Wienner's movement, whose theoretical values have been tabulated by Johansen and Juseluis, where $\hat{\lambda}_i$ denote the eigenvalues estimated under H_0 of the matrix $\breve{\Pi}_p$.

2.1.2. Maximum eigenvalue test

As a complementary to the preceding test, this cointegration test has no power to substitute it. In this regard, Johansen and Juselius (1990) put forward the following hypotheses:

$$\begin{cases} H_0: rg(\breve{\Pi}_p) = r \\ H_1: rg(\breve{\Pi}_p) = r+1 \end{cases} \quad (6)$$

To test the basic hypotheses, the authors decided to deploy the following statistic:

$LR = -T \log(1 - \lambda_{r+1}) \rightsquigarrow$Non-Standard Law, different from the above used one.

In accordance with the cointegration methodology prerequisites, the entirety of the considered variables is, a priori, endogenous. In this regard, Engle and Granger (1987) maintain that if one of the trace or maximum eigenvalue tests enables to identify at least one cointegration vector, there will be at least a single variable undergoing an adjustment process, and, therefrom, at least one causal relationship in a long-term relationship binding two variables X and Y. This theorem could be generalized to involve the case of N variables.

In this sense, Johansen (1992) developed a weak exogeneity test to highlight weakly exogenous variables, liable to "cause", correct "endogenous" or contribute to its adjustment, which culminated in setting up the Johansen (1992) weak exogeneity test.

2.1.3. Johansen's low exogeneity test (1992)

Devised by Johansen (1992), the "weak exogeneity test" serves to identify weakly exogenous variables within a long-term relationship. These variables help in correcting or adjusting non-weakly exogenous variables (adjusted towards an equilibrium level), i.e., undergoing an adjustment process towards their equilibrium level. For this purpose, it is necessary to administer a nullity test of the matrix parameters with adjustment coefficients α_ij. With respect to our study context, this test is implemented by considering the VECM (vectorial error-correction model) of order 1, as provided by equation 4, while assuming three-component involving vectors. It is worth reminding that in a three-variable system, there exist at most two cointegration vectors, wherein, at least one causal relationship should persist (Engle and Granger, 1987). In our context; the Johansen's weak exogeneity test is examined via anorder 3 VECM, such as:

$$\Delta Y_t = \breve{\Pi}_1 \Delta Y_{t-1} + \alpha\beta' Y_{t-1} \quad (7)$$

or, $\Delta Y_t = \Delta(Y_{1t}, Y_{2t} Y_{3t})' \; \alpha = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{pmatrix} \beta = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \\ \beta_{31} & \beta_{32} \end{pmatrix}$

The above VECM is rewritten as follows:

$$\Delta Y_t = \breve{\Pi}_1 \Delta Y_{t-1} + \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{pmatrix} \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \\ \beta_{31} & \beta_{32} \end{pmatrix}' Y_{t-1} \quad (8)$$

Considering the first difference of the first variable ΔY_{1t}, and based on the above figuring VECM, the following relation can be formulated:

$$\Delta Y_{1t} = \sum_{j=1}^{p-1} \Delta Y_{1t-j} + \alpha_{11}(\beta_{11} Y_{1t-1} + \beta_{21} Y_{2t-1} + \beta_{31} Y_{3t-1}) + \alpha_{12}(\beta_{12} Y_{1t-1} + \beta_{22} Y_{2t-1} + \beta_{32} Y_{3t-1}) + \epsilon_{1t} \quad (9)$$

Accordingly, the variable Y_{1t} is not weakly exogenous if the adjustment coefficient α_{11} is statistically non-zero, and vice versa. Thus, one could just administer a Student significance test to determine the non-weakly exogenous variable(s).

As for the variable Y_{2t}, the appropriate equation fit for testing whether it does, or does not, undergo an adjustment process turns out to be:

$$\Delta Y_{2t} = \sum_{j=1}^{p-1} \Delta Y_{2t-j} + \alpha_{21}(\beta_{11}Y_{1t-1} + \beta_{21}Y_{2t-1} + \beta_{31}Y_{3t-1}) + \alpha_{22}(\beta_{12}Y_{1t-1} + \beta_{22}Y_{2t-1} + \beta_{32}Y_{3t-1}) + \epsilon_{2t} \quad (10)$$

Testing the statistical significance of α_{22} amounts to testing whether the variable Y_{2t}does undergo an adjustment process, i.e., this test helps in determining whether $Y_{2t}$is weakly exogenous or not. Once α_{22} is discovered to be statistically zero, Y_{2t} turns out to be not weakly exogenous. In practice, the search for possible long-term relationships must be preceded by a VECM specification test. That is to say, it is a question of testing whether the VECM does enclose a constant vector, or not. Such a test should serve to recognize whether the variables in question do actually contain deterministic trends or not, as figuring below.

2.1.4. Testing the nature of the VECM specification

A priori, to ensure whether to introduce a constant vector into the VECM (case in which the relevant variables have a deterministic tendency), or a constant term associated with the time variable (t) (in this case, the variables in question have a quadratic tendency), the specification error should refer to administering a dummy Johansen test.

As a first step, we outline the test enabling to check whether the N variables appear to bear a deterministic tendency. Hence, the following hypotheses are put forward:

$$\begin{cases} H_0: variables\ Y_{it}\ have\ deterministic\ tendencies \\ H_a: \text{variables } \mathrm{Y_{it}} \text{ do not } havedeterministic\ tendencies \end{cases}$$

To test the null hypothesis, Johansen consider the following LR[2] statistic:

$$LR = -T \sum_{i=r+1}^{N} \log(\frac{1-\hat{\lambda}_i}{1-\dot{\lambda}_i}) \sim \chi^2_{N-r} \quad (11)$$

or, $\hat{\lambda}_i$: designates the estimated eigenvalues of $\breve{\Pi}_p$ under H_0; while: $\dot{\lambda}_i$: are estimated under H_a.

In a second step, we specify the test allowing to check whether the relevant variables do actually bear a quadratic trend, such as:

$$\begin{cases} H_0: \text{variables } \mathrm{Y_{it}} \text{ have quadratic tendencies} \\ H_a: \text{variables } \mathrm{Y_{it}} \text{ do not have quadratic tendencies} \end{cases}$$

To test the basic hypothesis, we appeal to the following statistic:

[2]LR: Likelihood Ratio or likelihood ratio. Depending on the selected threshold; this Johansen (1988) devised statistic helps in confirming the cointegration relationship between the variables or indices studied.

$$LR = -T\sum_{i=r+1}^{N}\log(\frac{1-\hat{\lambda}_i}{1-\dot{\lambda}_i}) \sim \chi^2_{N-r} \quad (12)$$

or: $\hat{\lambda}_i$: are the estimated eigenvalues of $\breve{\Pi}_p$ under H_0; while: $\dot{\lambda}_i$: are the values estimated underH_a.

In addition to the IRF methodology, we opt for applying the error correction modeling (ECM) methodology, to examine the fundamentals' impact on banking stress and vice versa. In what follows is a summary of this approach.

We assume a vector $Y_t = (Y_{1t,}Y_{2t,}Y_{3t,} \dots \dots . Y_{Nt,})'$ whose components are entirely integrated of order one $((Y_{it} \sim I(1))$, i.e., the first difference is stationary $(\Delta Y_{it} \sim I(0))$. We also assume the existence of at least a single long-term relationship among the components of Y_t, wherein,$\widetilde{U}_t$denotes the relationship emanating residual vector that refers to the deviations of a given 'endogenous' from equilibrium. Accordingly, a positive deviation indicates that the adjusted variable has increased more than necessary, while a negative deviation indicates that it has decreased more than necessary, thereby, developing a dynamic of deviations from the adjusted 'endogenous' equilibrium. Constructing an ECM model requires appealing to the representation theorem of Engle and Granger (1987). In their work, the authors demonstrated the existence of an equivalence relationship between the cointegrated system and ECM modeling, highlighting that once the system of variables Y_ts proves to be cointegrated, one could draw the following ECM representations:

$$\Delta Y_{1t} = \sum_{i=1}^{p}\alpha_{1i}\Delta Y_{1t-i} + \sum_{i=0}^{p}\alpha_{2i}\Delta Y_{2t-i} + \sum_{i=0}^{p}\alpha_{i3}\Delta Y_{3t-i} + \cdots . \sum_{i=0}^{p}\alpha_{iN}\Delta Y_{Nt-i} + \rho 1\widetilde{U}_{t-1} + V_{1t} \quad (13)$$

$$\Delta Y_{2t} = \sum_{i=1}^{p}\beta_{1i}\Delta Y_{2t-i} + \sum_{i=0}^{p}\beta_{2i}\Delta Y_{1t-i} + \sum_{i=0}^{p}\beta_{i3}\Delta Y_{3t-i} + \cdots . \sum_{i=0}^{p}\beta_{iN}\Delta Y_{Nt-i} + \rho 2\widetilde{U}_{t-1} + V_{2t} \quad (14)$$

$$\Delta Y_{Nt} = \sum_{i=1}^{p}\gamma_{Ni}\Delta Y_{Nt-i} + \sum_{i=0}^{p}\gamma_{2i}\Delta Y_{2t-i} + \sum_{i=0}^{p}\gamma_{3i}\Delta Y_{3t-i} + \cdots . \sum_{i=0}^{p}\gamma_{1i}\Delta Y_{1t-i} + \rho N\widetilde{U}_{t-1} + V_{Nt} \quad (15)$$

2.2. The Impulse Response Functions (SIMS) method

The Keynesian approach helps in simulating a particular economic system's internal dynamics by shocking an exogenous variable. Contrarily, however, SIMS (1980) considers that in a particular economic system, all variables are endogenous. Thus, simulation of a certain economic policy is performed by carrying out shocks on the error term of the endogenous variable to simulate.

Thus, to simulate a given economic system's dynamics conformingly with the SIMS approach, the following structural VAR should be assumed:

$$BY_t = B_1Y_{t-1} + B_2Y_{t-2} + B_2Y_{t-2} + \cdots \dots B_pY_{t-p} + U_t \quad (16)$$

Where: the vector Ut admits a matrix of orthogonal variances-covariances. In effect, incorporating orthogonal shocks makes it possible for us to actually simulate a given policy, since reduced shocks are expressed as a linear combination of orthogonal shocks, thereby, bearing no economic significance.

Implementation of the SIMS method requires reformulating equation (16) under the form of structural shocks. To this end, we appeal to Wald's (1954) theorem, enabling to translate a reduced VAR into reduced shocks multiplied by the so-called "matrices impact". Thus, we assume the vector Y_t to follow a stable reduced VAR of order p, such as:

$$Y_t = A_1 Y_{t-1} + A_1 Y_{t-2} + A_1 Y_{t-2} + \cdots \ldots A_1 Y_{t-p} + \in_t.$$

Following Wold's theorem (1954), this system is rewritten as:

Y_t=C(L)$\in_t$, wherein, r C(L) is a polynomial of impact matrices, and $\in_t$ signifies reduced shocks. This equation can be formulated as follows:

$$Y_t = C_0 \in_t + C_1 \in_{t-1} + C_{20} \in_{t-2} + C_3 \in_{t-3} + \cdots \ldots \ldots C_p \in_{t-p} \quad (17)$$

As the chocks $\in_t$are not structural,it is necessary to reformulate the equation to incorporate reduced shocks. Since $\in_t = B^{-1} U_t$, we can modify equation (17) in such a way as to exclusively involve structural shocks:

$$Y_t = C_0 B^{-1} U_t + C_1 B^{-1} U_{t-1} + C_2 B^{-1} U_{t-2} + C_3 B^{-1} U_{t-3} + \cdots \ldots \ldots C_p B^{-1} U_{t-p} \quad (18)$$

On maintaining $C_0 B^{-i} = E_i$, the equation turns out to be:

$$Y_t = E_1 U_t + E_1 U_{t-1} + E_2 U_{t-2} + E_3 U_{t-3} + \cdots \ldots \ldots E_p U_{t-p} \quad (19)$$

Thereby, shifting from the structural model (16) to the reduced VAR:

$$Y_t = B^{-1} B_1 Y_{t-1} + B^{-1} B_2 Y_{t-2} + B^{-1} B_2 Y_{t-2} + \cdots \ldots B^{-1} B_p Y_{t-p} + B^{-1} U_t \quad (20)$$

3. Econometric modeling of the relationship between bank stress index and macroeconomic variables

In a first stage, this section is devoted to exploring the impact of shocks emanating from a number of macroeconomic variables (the GDP growth rate (GDP), current account as GDP percentage (SCP), inflation rate (INF), budgetary balance as GDP percentage (SBP), external debt rate (DTex), export growth rate (EXP)) on banking stress index. In a second stage, we proceed with analyzing these macroeconomic variables associated dynamics, following a shock, on the banking stress index. For this purpose, and as already stated, we consider applying the SIMS method (1972) to estimate the impulse functions of the cited macroeconomic variables as well as the bank stress index, following positive orthogonal shocks. Additionally, to validate our attained empirical results, appeal is made to ECM modeling techniques.

3.1. Impulse Response functions: SIMS method

At this level, we proceed with estimating the reaction functions enabling to explain the dynamics of GDP, growth of EXP, SCP, SBP, TINF, DTex and IS, following a positive innovation affecting the bank stress index as well as the investigated macro-economic variables. Estimation of these impulse functions has been implemented via the Cholesky decomposition approach, and the reached empirical results are depicted in Figure 2.

Hence, a positive structural shock on the bank stress index might further worsen the country's economic situation, while a positive shock on the examined macroeconomic aggregates could help in noticeably boosting economic and financial indicators, such as the banking stress index. Our achieved empirical results appear to reveal that following a structural shock, the investigated macroeconomic variables turn out to demonstrate a deterioration from their initial state: a decline on the horizon h regarding exports, GDP, higher inflation along with current and budgetary balances. Estimates of the different IRF functions reveal the significance of the GDP "striking" impulse coefficients, which is not so much the case for exports, wherein, coefficients prove to be predominantly significant during the first periods. The current and budgetary balances related IRF functions' dynamics remain timid, displaying significant momentum coefficients. As to the inflation and external debt associated reaction functions, they tend to demonstrate a significant response of both macroeconomic variables, though, with non-significant pulse coefficients.

Similarly, our empirical results appear to demonstrate a positive shock on inflation, external debt as well as budgetary and current account balances, which helped only increase bank stress, as highlighted through the table 4, where the impulse coefficients turn out to be statistically significant for both of the budgetary and current balances, which is not the case for the inflation and external debt IRFs. Our empirical results display also a drop in banking stress index following a positive shock on exports, to experience an increase in its level following a positive shock on inflation. Noteworthy, however, is that the impulses estimated in regard to these cases tend to remain globally insignificant. It is also worth mentioning that following shocks, the bank stress index associated dynamics are discovered to stabilize in the long term whether decreasingly or increasingly. Initially, a major important result attained following this impulse analysis turns out to be the noticeable sensitivity of the banking stress index to positive shocks on GDP, reflected by a significant drop in impulse coefficients, to record negative values approaching -4. This is not the case for exports, where stress index proves to register insignificant drops, with estimated impulse rates of around -0.5. A second main result ensuing following this empirical exploration lies ina significant increase in banking stress index following a positive shock on Tunisia's external debt, even though the impulse coefficients appear to remain weakly significant. Still, this index seems to be experiencing a timid and significant increase following a positive shock on inflation as well as on budgetary and current account balances.

In sum, one could well conclude that, on the one hand, a positive orthogonal shock on the banking stress index could only help in further aggravating the country's macroeconomic situation. In effect, our empirical results highlight a noticeable decline persistent in both of the GDP and exports' growth rates. Noteworthy, also, are the impulses' significantly important coefficients that have continued to "strike" the GDP dynamics, which is not the case for exports. Similarly, this stress index shock has also weakened the country's economy noticeably, resulting in an increased external debt and inflation in Tunisia. On the other hand, it has been discovered that the banking stress index proved to respond timidly or weakly to the GDP and exports associated positive shocks (see IRF above), while witnessing a significant deterioration following the external debt, inflation as well as current and budgetary balances shocks affecting. Based on this empirical analysis of the bank stress index impulse functions, one could wellnote that this index appears to stand as highly sensitive to any positive shock on any stress amplifying variable, unlike the macroeconomic variables subject of our empirical study.

3.2. Analysis of the bank stress index and macroeconomic variables binding relationship

This section is devoted to determining the impact of macroeconomic variables on banking stress index, examined through an error correction modeling framework. Initially, we proceed with highlighting the model's theoretical specification:

$$\Delta IS_t = \alpha_0 + \sum_{i=1}^{3} \beta_{1i} \Delta IS_{t-i} + \sum_{i=0}^{3} \beta_{2i} \Delta EXP_{t-i} + \sum_{i=0}^{3} \beta_{3i} \Delta PIB_{t-i} \sum_{i=0}^{3} \beta_{3i} \Delta INF_{t-i} + \sum_{i=0}^{3} \beta_{4i} \Delta SBP_{t-i} + \sum_{i=0}^{3} \beta_{5i} \Delta SCP_{t-i} + \delta\, ECT_{t-1} + U_t \quad (21)$$

In accordance with Engle and Granger (1987), this specification presumes the existence of a long-term relationship or cointegration among the variables: IS (bank stress index), EXP (export growth rate), GDP (GDP growth rate), INF (inflation growth rate, SCP (current balance to GDP ratio) and SBP (budget balance to GDP ratio). Prior to verifying the existence of a long-term relationship between these variables, we undertake to determine their binding integration degree. Actually, administration of both of the ADF and KPSS stationarity tests highlighted well that the entirety of these variables turn out to be stationary in prime difference, while proving to be non-stationary as to the relevant level.

Such an empirical result allows for applying the cointegration test of Johansen-Juselius (1990). Table 1 indicates that the Johansen's procedure demonstrates well the persistence of a single long-term relationship binding the macroeconomic variables figuring on the table.

The administered Johansen's cointegration (trace) test reveals the persistence of two long-term relationships binding the bank stress index and a number of economic fundamentals, namely, the exports (EXP) and gross domestic product (GDP)growth rates, inflation rate (INF), exchange rate (TCH) along with the balance of payments deficit rate (SBP) (see the annex: table 2). Further to the econometric test, identifying the long-term relationships also requires setting up an economic criterion, i.e., seeking long-term relationships displaying adequate signs, asset by the economic theory. Hence, we consider estimating the following bank stress index long-term relationship:

$$\widehat{IS}_t = -0.15EXP_t - 0.38PIB_t + 0.91INF_t + 2.5TCH_t + 0.48SBP_t \qquad (22)$$

Accordingly, one could compute the banking stress associated deviations based on equilibrium value noted $\widehat{U}_t = IS_t - \widehat{IS}_t$, as illustrated through figure 3.

The above empirical results, relating to the existence of a long-term relationship, help in reformulating the ECM modeling theoretical specification of bank stress index, whose OLS estimation yields the relevant econometric result. Following:

$$\Delta IS_t = 0.00268 - \underset{(-5.47)}{0.0938\Delta EXP_t} - \underset{(-3.16)}{0.0508\Delta EXP_{t-2}} - \underset{(-3.05)}{0.1475\Delta PIB_t} - \underset{(-2.095)}{0.096\Delta PIB_{t-2}}$$

$$+\underset{(4.779)}{0.451\Delta INF_t} - \underset{(-2.808)}{0.1958\Delta SBP_t} + \underset{(1.79)}{0.65\Delta TCH_t} - \underset{(-1.71)}{0.16ECT_{t-1}} \qquad (23)$$

$R^2=0.77$ DW=2.44

The achieved empirical results reveal well that the entirety of banking stress index explanatory variables tend to bear, or ‘carry’, estimators conveniently coinciding with economic logic. Thus, an increase in the exports and GDP growth rate results in a drop in banking stress index. As highlighted in the estimated equation, such an effect proves to be rather noticeably pronounced with respect to the variable GDP than EX, a result that conforms and reflects well the actual economic reality. More importantly, also, is that both of the variables associated estimators turn out to be highly significant, noticeably confirming our economic interpretation regarding both fundamentals’ (GDP and EXP) short-term effect on bank stress index. The banking stress related variable proves to undergo a noticeable adjustment process, as reflected by the significant "ECT" variable associated adjustment coefficient. Indeed, controlled through weakly exogenous variables (EXP, GDP, INF and SBP, see appendix), this adjustment process proves to be exclusively weakly significant, as only 16% of banking stress mismatch will be corrected each semester, relevant to the equilibrium.

Noteworthy, also, is that the attained empirical results reveal well that the short and long term effects appear to differ remarkably. Apart from the exports’ effect, the variables

GDP, INF and TCH turn out to display varying long-term effects on banking stress index than short-term ones. In effect, the above figuring long-term relationship highlights well that a 1% increase in exports and gross domestic product helps in reducing the bank stress index by 0.15 and 0.3 points over the long term, respectively, while a1% increase in the inflation and unemployment rates are likely to bring about 1.5 and 3.9 points long-term increases in banking stress, respectively. These empirical results show that the deteriorating banking situation in Tunisia has occurred gradually, and has taken several years to materialize significantly. A comparison of the short and long run effects confirms well our concluded findings, as highlighted through table 3.

It is worth mentioning, however, that our implemented empirical work has been conducted to examine the banking stress index and economic fundamentals binding relationship as exclusively observed form a single perspective, or direction, heading from Economic fundamentals towards banking stress, following the Granger (1969) causality test logic and line of thought. Accordingly, the test administration ensuing empirical results ended up outlining that the investigated economic fundamentals proved to stand at the origin of banking stress, as illustrated through table 5 (see appendix).

Overall, the administered Granger causality test turned out to reveal that it is actually the fundamentals that brought about the banking index stress, rather than the inverse. Additionally, it also revealed the existence of a double causality between exports and banking stress, on the one hand, and between inflation and banking stress, on the other.

4. Empirical Results and Conclusions

The present study is elaborated to investigate the connection between banking stress and economic fundamentals in regard to the Tunisian context, observed over the time period ranging from 1998 to 2021 on semi-annual data basis. It is predominantly focused on determining the impact of a selection of fundamentals (e.g., exports, gross domestic product, inflation, exchange rate and balance of payments) on banking stress index. For this aim, we initially proceeded with determining the examined variables integration degree through administration of two stationarity tests (ADF and KPSS). Actually, it has been discovered that the variables, subject of study, turned out to be non-stationary in level, while being stationary in first difference, which made us appeal to an error correction model (ECM). In this respect, however, Engle and Granger (1987) demonstrated an equivalence between ECM modeling specification and the existence of cointegration relationship(s). More specifically, the Johansen-Juselius (1990) devised methodology allowed us to empirically demonstrate the persistence of three long-term relationships binding the variables: IS, EXP, GDP, INF, TCH and SBP. Actually, identification of these relationships allowed us to exclusively pinpoint a single fit long-term relationship whose variables proved to bear, or ‘carry’, estimators with signs conforming to economic logic. At this level, the maintained long-term relationship enabled to reveal that the bank stress index appeared to stand as very elastic in respect of inflation and exchange rate, insofar as 1% increases in inflation as well as exchange ratesare liable to result in long-term increases in bank stress by 1.5% and 3.9%, respectively. However, this stress index appears to be non-elastic, in the long run, with regard to exports, gross domestic product and balance of payments, since a 1% increase in exports and GDP is likely to engender long-term falls in banking stress index by just 0.15% and 0.3%, respectively. As for the SBP, a 1% increase in its long-term level helps in bringing about a 0.19% increase in Tunisia's banking stress. As a matter of fact, the ECM modeling estimation highlights well the presence of an appropriate adjustment process for this index, providing significantly negative adjustment coefficient. Nonetheless, this adjustment design remains relatively small in scope, as a stress index deviation from its equilibrium level could yield an adjustment of 0.17% to this imbalance. More importantly, however, it is worth noting that the correction modeling estimation of banking stress index in Tunisia proved to demonstrate an absolutely non-sensitive short-term variation to the various adjustment variables investigated, specifically; exports, GDP, inflation, exchange rate and balance of payments deficit, as illustrated on table 4.

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Annex

Table 1: ADF and KPSS test

Indices	ADF test		KPSS Test	
	In level	**Primary Difference**	**In level**	**Primary Difference**
IS	0.16	-3.35	0.77	0.46
EXP	-0.88	-6.92	0.62	0.32
PIB	-1.84	-6.49	0.65	0.20
INF	0.05		0.83	0.50
SBP	-1.52	-3.77	0.64	0.24
SCP	-2.19	-6.12	0.65	0.11

Table 2: Cointegration Relationship between Banking Stress Index and Fundamentals (Unrestricted Cointegration Rank Test – Tract)

	Statistic L	Critical value (5%)	Decision
H0 : Rang $\hat{\pi} \leq 0$ Ha : Rang $\hat{\pi} \geq 1$	204,7	95,75	Rejection of H0
H0 :Rang $\hat{\pi} \geq 1$ Ha : Rang $\hat{\pi} \geq 2$	112.48	69,81	Rejection of H0
H0 :Rang $\hat{\pi} \geq 2$ Ha : Rang $\hat{\pi} \geq 3$	55.41	47,87	Rejection of H0
H0 :Rang $\hat{\pi} \geq 3$ Ha : Rang $\hat{\pi} \geq 4$	16.70	29,7	Accept H0

Note: Trace test indicate 3 cointegrating equations at the 0.05 level. * denotes rejection of the hypothesis at the 0.05 level. ** MacKinnon-Haung-Michelis (1999) p-values.

Table 4: Short and long term effects on Tunisia's banking stress

Variables	Long-term effect (in points)	Short-term effect (in points)
Export (%)	-0.15	-0.143
GDP (%)	-0.30	-0.22
Exchange rate (%)	3.9	0.46
Inflation (%)	1.5	0.65
Budgetary balance.	0.19	0.07

Table 5: Granger Causality Test: Bank Stress-Economic Fundamentals

Causality			Fisher Statistic	Causal Direction Offset
DEXP	→	DIS	1.26	Yes
DIS	→	DEXP	3.31	yes
DPIB	→	DIS	9.11	yes
DIS	→	DPIB	0.16	No
DINF	→	DIS	3.9	Yes
DIS	→	Dinf	12.3	yes
DTCH	→	DIS	10.1	yes
DIS	→	DTCH	0.14	No
DSCP	→	DIS	2.88	yes
DIS	→	DSCP	1.77	YES

Figure 1: Common trend of IS, EXPORT and inflation in Tunisia

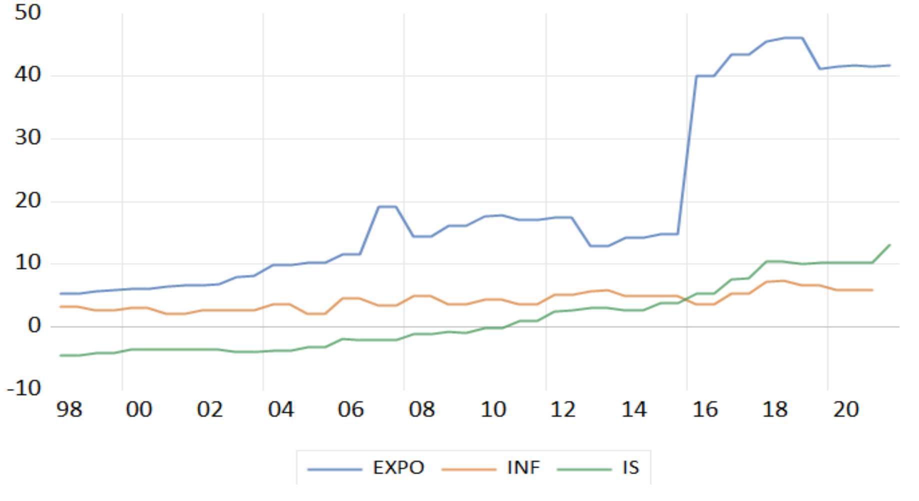

Figure 2: IRF (Impulse Response Function) functions of bank stress index and selected macroeconomic variables

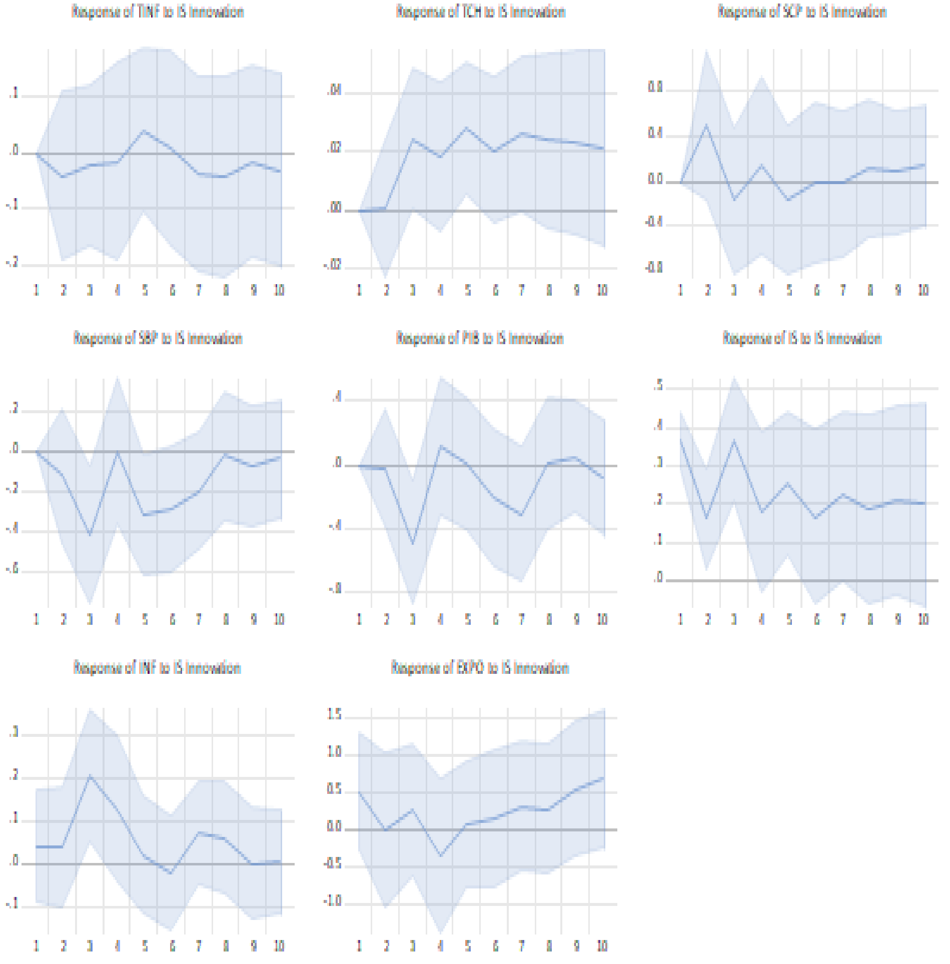

Figure 2 continued

Figure 2 continued

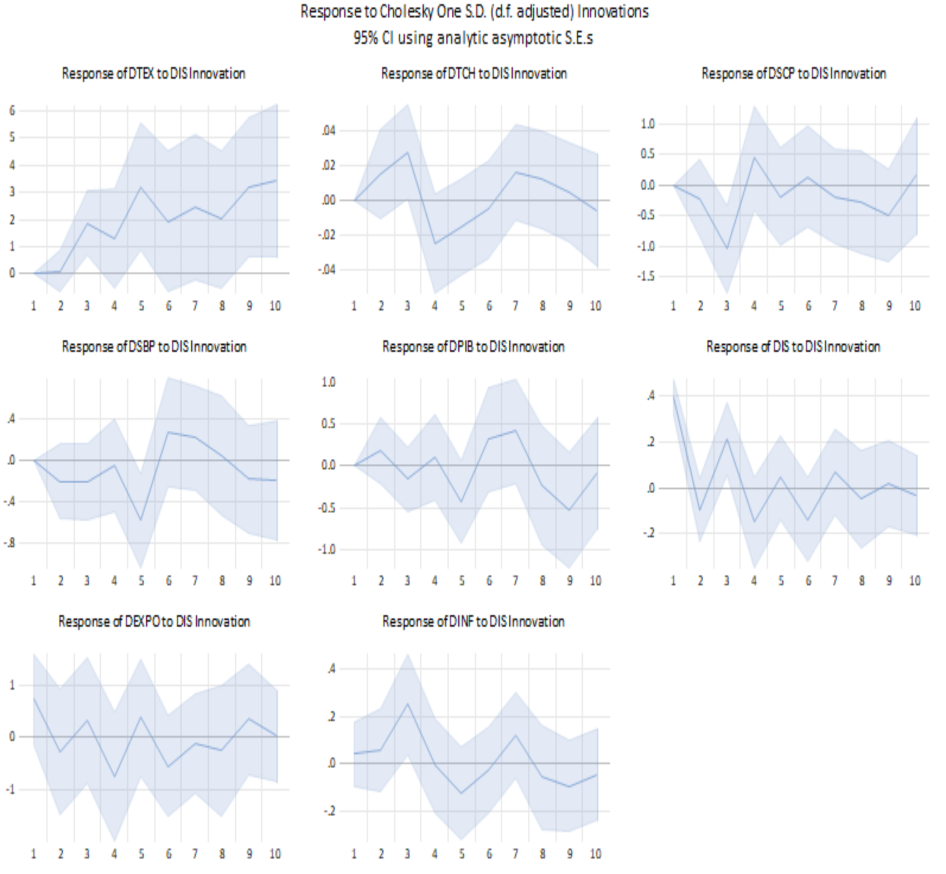

Figure 2 continued

Figure 3: Deviations of banking stress index in Tunisia from its equilibrium value