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Asymmetric Information and Equilibrium Price

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Abstract. In this short note, Kyle's results are extended. Indeed, it is seen that a class of equilibrium prices exists, called it Kyle's equilibrium class. To this end, first, it is proved that the proportion of variance of price suggested by Kyle (1985) over the variance of size of asset traded by noise trader is eight. Then, this number (eight) is replaced with a positive known parameter. By maximizing the profit function of informed trader with respect to mentioned parameter, some conditions are derived and the Kyle's equilibrium class is derived. Simulation-based results are obtained.

Keywords: Asymmetric Information, Class of Equilibrium, Kyle's Equilibrium, Normal Distribution

1. Introduction

Asymmetric information means that one party has more or better information than the other when making decisions and transactions. The imperfect information causes an imbalance of power. For example, when you are trying to negotiate your salary, you will not know the maximum your employer is willing to pay and your employer will not know the minimum you will be willing to accept (see Brown *et al*., 2004). Accurate information is essential for sound economic decisions. When a market experiences an imbalance it can lead to market failure. Asymmetric information causes an imbalance of power. (see Izquierdo and Izquierdo , 2007). Adverse selection is a term used in economics that refers to a process in which undesired results occur when buyers and sellers have access to imperfect information, also known as asymmetric information (see Lambert *et al*., 2012).

In this paper, a new class for Kyle equilibrium is derived. First, by some algebraic and statistical manipulations, the result of Kyle (1985) is simply extended. Indeed, it is concluded that the proportion of variance of risk neutral price suggested by market maker (result of Kyle) over to the variance of size of asset traded by noise trader is eight. To extend the result of Kyle (1985), the above variance and the variance of size of traded by noise trader are transformed by scale factors α^2 and β^2, respectively. By maximizing the profit of informed trader with respect to α and β, some mathematical conditions on α and β are derived and then, the new Kyle's equilibrium class of prices is derived. The rest of paper is organized as follows. In the next section, the main theoretical results are given. Some simulation results are given in section 3. Conclusions are given in section 4.

2. Main Result

Using the Kyle's notations, the price of asset is V which is normally distributed with mean $\mu_v$and σ_v^2, that is $V \sim \mathrm{N}(\mu_v, \sigma_v^2)$, where U and V are independently distributed. The noise trader trades$\mathrm{U} \sim \mathrm{N}(0, \sigma_\mathrm{u}^2)$ size of asset and the informed trader decisions about $X = \mathrm{aV} + \mathrm{b}$ size of asset, where under equilibrium case, then:

$$a = \frac{\sigma_u}{\sigma_V} \text{and } b = -\frac{\sigma_u}{\sigma_V}\mu_v$$

The price maker suggests the risk neutral price is $p = p(y) = E(V|Y = y) = cy + d$,where under equilibrium condition, then$c = 2\frac{\sigma_V}{\sigma_u}, d = \mu_v$ and $Y = X + U$.

2.1. Some Results

Notice that $E(X) = 0, var(X) = var(U) = \sigma_u^2$. Thus, it is easy to see that $E(Y) = 0$ and $var(Y) = 2\sigma_u^2$. Also, $E\big(p(Y)\big) = \mu_v$ and $var\big(p(Y)\big) = 8\sigma_u^2$. The following proposition states that the equilibrium conditions are derived using assuming simple conditions like $var(X) = \sigma_u^2$ and $var\big(p(Y)\big) = 8\sigma_u^2$. Also, the statistical distribution of informed trader profit is derived.

Proposition 1: If $E(X) = 0, var(X) = \sigma_u^2$ and $E\big(p(Y)\big) = \mu_v$ and $var\big(p(Y)\big) = 8\sigma_u^2$, then ($a$) the coefficients of equilibrium p and X is obtained; and (b) The profit $\pi = -\frac{\sigma_u}{\sigma_V}(\mathrm{V} - \mu_v)^2$ is distributed as:

$$-\sigma_u\sigma_v\chi^2_{(1)}$$

with mean and variance $-\sigma_u\sigma_v$, $2\sigma_u^2\sigma_v^2$, respectively.

2.2 Motivation

This fact motivates author to let similar conditions and to derive the class of equilibrium conditions. As follows, the class of equilibrium cases is characterized. To this end, suppose that:

$$E(X) = 0, var(X) = \alpha^2\sigma_u^2,$$

and

$$E\big(p(Y)\big) = \mu_v, var\big(p(Y)\big) = \beta^2\sigma_u^2$$

for some real numbers α and β.

Proposition 2: Under the new equilibrium conditions, i.e., $var(X) = \alpha^2\sigma_u^2$ and $var\big(p(Y)\big) = \beta^2\sigma_u^2$, then

$$(a)\ a = \alpha\frac{\sigma_u}{\sigma_v},\ b = -a\mu_v,\ c = \frac{\beta}{\sqrt{1+\alpha^2}} \text{ and } d = \mu_v$$
$$(b)\ \pi = a(1-ac)(\mathrm{V}-\mu_v)^2 = \alpha\left(1-\frac{\sigma_u}{\sigma_v}\frac{\alpha\beta}{\sqrt{1+\alpha^2}}\right)\sigma_u\sigma_V Z^2$$

where Z is standard normal random variables.

2.3 Results

There are many solutions to maximize π with respect to α and β.
Let $f(\alpha,\beta) = \alpha\left(1-\frac{\sigma_u}{\sigma_v}\frac{\alpha\beta}{\sqrt{1+\alpha^2}}\right)$

Then, $\frac{\partial f}{\partial \alpha} = 0$ implies that:

$$\beta = \frac{(1+\alpha^2)^{\frac{3}{2}}}{(\alpha^2+2)\alpha}\frac{\sigma_V}{\sigma_u}$$

The above results are summarized in the following theorem.

Theorem 1: The new class of Kyle equilibrium is given by

$$\begin{cases} p = p(y) = E(V|Y=y) = cy + d \\ Y = X + U \\ \beta = \frac{(1+\alpha^2)^{\frac{3}{2}}}{(\alpha^2+2)\alpha}\frac{\sigma_V}{\sigma_u} \end{cases}$$

where for suitable choice of α, corresponding values for β's are given. Here, $c = \frac{\beta}{\sqrt{1+\alpha^2}}$ and $d = \mu_v$.

Remark 1: As special case, suppose that $f(\alpha) = \sqrt{1+\alpha^2}$

Then $\quad \pi = kf(\alpha) = \alpha\left(1-\frac{\sigma_u}{\sigma_v}\alpha\right),$

where k is positive number.

This function has maximum at $\alpha = \frac{\sigma_V}{2\sigma_u}$. Then, $a = 0.5, b = -0.5\mu_v, c = 1$ and $d = \mu_v$. By this selection, $f(\alpha,\beta) = \frac{\alpha}{\alpha^2+2}$. Then, $\frac{\partial f}{\partial \alpha} = 0$ makes the optimum α is $\sqrt{2}$. In this way, $\beta = \sqrt{\frac{27}{32}\frac{\sigma_V}{\sigma_u}}$.

4. Conclusions

Kyle (1985) constructed a model to exploit monopoly power in the market. His work is extended in two ways. Some of researchers such as Duffie (1985), Back (1992*a*), Aksamit and Hou (2016) extended the problem proposed by Kyle by changing his model. For example, they considered the Black-Scholes model of asset pricing for traders. Some others like Back (1992*b*) applied the model of Kyle for financial modeling such as option pricing.

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