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Stratified Market Equilibria: An Application of Maximal Overlap Discrete Wavelet Analysis

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Abstract: Unlike mainstream financial analysis, our research suggests that the market which trades a single stock is actually stratified. For a given stock, the expected return in each stratified market forms a unique relationship with the expected market return. The revealing of such a multi-level market structure challenges the well-established capital asset pricing hypothesis and the prevailing equilibrium analysis. To stratify a market which trades a single financial asset, we rely on the maximal overlap discrete wavelet transform (MODWT). Akaike and Bayesian information criteria are applied for selecting the appropriate model to reveal the relationship between stock returns and market returns in each stratified market. Monte Carlo experiments are then used to verify the consistency of an estimator. Our study has significant theoretical implications. If a market for a single asset can be stratified, it begs the question whether a modified or alternative theory is needed for explaining financial market activities. More significantly, our analysis raises the issue of limited usefulness of the concept of equilibrium in financial economics. If the expected asset return on a financial instrument is actually composed of several components, is the equilibrium return made of several equilibrium components or an aggregation of stratified market disequilibria?

Keywords: Market Stratification, MODWT, CAPM, Market Equilibrium

JEL Classification Number: G1

1. Introduction

Financial economists are often interested in the relationship between asset returns and market returns. The well-established and widely accepted paradigm is the capital asset pricing model (CAPM). This model has been challenged by behavioral economists on the grounds of bounded rationality and heterogeneous agents (Barberis and Thaler, 2003). Beyond laboratory experiments, economists have also been able to construct analytically tractable models, known as the heterogeneous agent models (HAMs), to simulate the behavior of heterogeneous agents with bounded rationality (Hommes, 2006; LeBaron, 2006).

In all previous studies, economists and psychologists investigate investment behavior over time, and the market for trading any single asset is not further divided into submarkets. If investors are not fully rational and heterogeneous, they may apply different strategies and criteria to trade the same type of instruments and hold these instruments for various lengths of time. Due to the limited availability of data, conventional time series analysis does not allow a researcher to investigate trading frequencies. This research suggests an alternative. When investment behavior is examined in both time and frequency dimensions, we are able to stratify a market for trading any single asset into multiple layers. In each stratified market, the expected return on an asset is correlated with an expected segmented-market return.

In spectrum analysis, long term variations are associated with low frequency waves, while short-term fluctuations are captured by high frequency waves. Fourier analysis has often been criticized because it only captures the relationship between variables in terms of frequencies. To compensate, windowed Fourier analysis has been used to capture the changes over time. This technique, however, has also been faulted on its incapability to associate frequencies with varying time frames. Wavelet analysis, on the other hand, is capable of analyzing frequencies associated with various time scales. That is, when a wavelet has a short time support, it allows an analyst to examine a short-lived phenomenon; if a wavelet has a long time support, it can be used to study a long lasting periodic behavior. Thus, wavelet analysis has appealing statistical advantages over traditional techniques.

Market stratification is possible because the multi-resolution analysis of wavelets provides a natural framework for analyzing frequency patterns associated with different time scales. To remove the issue of sensitivity to the starting and ending point of a series, the technique of maximal overlap discrete wavelet transfer (MODWT), as suggested by Percival and Walden (2000), is used in this study.

Over the years, natural and social scientists have paid growing attention to wavelet analysis. By providing a comprehensive theoretical framework, Daubechies (1992) lays the foundation of wavelet analysis. A detailed review of wavelets and filter banks is given by Strang and Nguyen (1996). Donoho and Johnstone (1994) and Gao (1997) broaden the applicability of wavelet theory to analyze time series with non-stationary process and long memory. Using wavelet coefficients to estimate the spectral density, Neumann (1996) is able to derive an asymptotic normal distribution from a non-Gaussian process. Priestley (1996) suggests the use of wavelet analysis to develop evolutionary spectra for time-dependent spectral analysis. A comprehensive review of wavelet methods for time series analysis is given by Percival, and Walden (2000). Addressing technical needs, Press, et al. (2007) provide numerical recipes for scientific computing. Mallat (2008) provides a review of the multi-resolution analysis (MRA) and wavelet transform modulus maxima method, among other fundamental concepts, and their applications in signal processing.

Ramsey (1999) demonstrates wide applicability of wavelet theory in economics and finance. Using compactly supported wavelets, Jensen (1999, 2000) applies a maximum likelihood method to estimate long memory parameters. Lee and Hong (2001) develop a widely applicable wavelet-related consistency test for serial correlation. The usefulness of wavelet filter methods in these fields is also shown by Gençay, Selçuk and Whitcher (2001). Nason and Sapatinas (2002) use the wavelet packet transfer function to model non-stationary time series. In and Kim (2013) offer a thorough review of the wavelet theory in finance. He and Lin (forthcoming) provides a bird's eye view of wavelet filter methods in economics and finance.

The arrangement of this article is as follows. Section 2 shows a discrete ARMA and MODWT model and investigates the properties of the wavelet coefficient. Relying on the Whittle approach, AIC and BIC are used for choosing the competing models associated with an arbitrary level decomposition. Section 3 presents the Monte Carlo experiments to examine the power of the test. Section 4 applies our model to analyze the stock returns of Yulon Motor Company, and to measure~~s~~ stratified systematic risks. Section 5 offers concluding remarks that summarize this article's contributions.

2. Wavelet-based Spectral and Spectral Density Functions

To investigate the risk-return relationship, we incorporate an ARMA process and spectrum into wavelet analysis. Consider a time series $\{u_t\}$ with a stationary ARMA (p,q):

$$\phi(L)u_t = \theta(L)a_t , \tag{1}$$

where L is the lag operator, and a_t is white noise with variance σ^2. Terms $\phi(L) = (1-\phi_1 L - ... - \phi_p L^p)$ and $\theta(L) = (1-\theta_1 L - ... - \theta_q L^q)$, respectively, represent the AR and MA polynomial operator functions with roots outside the closed unit circle.

Following Daubechies (1992) as well as Percival and Walden (2000), the wavelet and scaling coefficients resulting from the maximal overlap discrete wavelet transform

(MODWT) at level j are:

$$\tilde{w}_{j,t}=\sum_{\ell=0}^{N_j-1}\tilde{h}_{j,\ell}u_{t-\ell \bmod T} \text{ and } \tilde{v}_{j,t}=\sum_{\ell=0}^{N_j-1}\tilde{g}_{j,\ell}u_{t-\ell \bmod T}, \quad t = 1,\ldots, T,$$

where $N_j=(2^j-1)(N-1)+1$, with N as the length of the filter, $\tilde{h}_{j,\ell}=2^{-j/2}h_{j,\ell}$; $\tilde{g}_{j,\ell}=2^{-j/2}g_{j,\ell}$, with $h_{j,\ell}=\sum_{k=0}^{N}h_k g_{j-1,\ell-2^{j-1}k}$; and $g_{j,\ell}=\sum_{k=0}^{N}g_k g_{j-1,\ell-2^{j-1}k}$ with $\ell = 0,\ldots, N_j - 1$. If $j = 1$, then $h_{1,j}=h_j$ and $g_{1,j}=g_j$. Note that, for MODWT, the wavelet coefficient, $\tilde{w}_{j,t}$, results from the application of a mother wavelet to a time series at level j centered at time t, which can be used to detect the occurrence of an event at a specific point of time.

In spectrum analysis, Whittle method is often considered as a generalized and efficient technique of parameter estimation. For our purpose, the use of wavelet analysis necessitates the development of an applicable Whittle method. Unlike the traditional Whittle estimator associated with periodogram covering the whole length of a time series, the Whittle estimator used here must allow us to investigate the relationship among periodograms associated with short as well as long time periods. Furthermore, the Whittle estimator based on MODWT must also be stable, efficient and consistent.

As scale level j increases, the associated time scale becomes longer and the frequency of each interval decreases. Figure 1 shows the spectra of wavelet coefficients at different levels, based on the Daubenchies recipe with $N = 8$. The superimposition of these figures on each other offers a hint on the relationship among the spectra, and therefore on the Whittle estimator. The transform weight matrix, according to Percival and Walden (2000), can be constructed by letting $W = Pu$, or

$$\tilde{W}=\begin{bmatrix}\tilde{V}_J\\ \tilde{W}_J\\ \vdots\\ \tilde{W}_j\\ \vdots\\ \tilde{W}_2\\ \tilde{W}_1\end{bmatrix}=\begin{bmatrix}P_{V,J}\\ P_{W,J}\\ \vdots\\ P_{W,j}\\ \vdots\\ P_{W,2}\\ P_{W,1}\end{bmatrix}u=\begin{bmatrix}\tilde{A}_J\tilde{A}_{J-1}\cdots\tilde{A}_1\\ \tilde{B}_J\tilde{A}_{J-1}\cdots\tilde{A}_1\\ \vdots\\ \tilde{B}_j\tilde{A}_{j-1}\cdots\tilde{A}_1\\ \vdots\\ \tilde{B}_2\tilde{A}_1\\ \tilde{B}_1\end{bmatrix}u=Pu$$

where $P_{V,J}$ and $P_{W,j}$, $j = 1,\ldots, J$, are $T \times T$ matrices, and T is the sample size. Thus, P is a

$\{(J + 1)\ T\} \times T$ matrix, with the scaling coefficients $\tilde{V}_J = (\tilde{v}_{J,1}, \tilde{v}_{J,2}, \cdots, \tilde{v}_{J,T})'$, and wavelet coefficients $\tilde{W}_j = (\tilde{w}_{j,1}, \tilde{w}_{j,2}, \cdots, \tilde{w}_{j,T})'$. Note that $P'P = I_T$ and $y = P'\tilde{W}$. The original series can be recovered by $u = P'_{V,J}\tilde{V}_J + P'_{W,J}\tilde{W}_J + \cdots + P'_{W,1}\tilde{W}_1$. Alternatively, we have

$$u = \tilde{S}_{y,J} + \sum_{j=1}^{J} \tilde{D}_{y,j}\ , \tag{2}$$

where $\tilde{S}_{y,J} = P'_{V,J}\tilde{V}_J$, $\tilde{D}_{y,j} = P'_{w,j}\tilde{W}_j$ for $j = 1$ to J, and $u'u = \tilde{V}_J'\tilde{V}_J + \sum_{j=1}^{J}\tilde{W}_j'\tilde{W}_j$. If the covariance matrix of u is Ω, then the quadratic form used to construct the maximal likelihood function by vector y becomes

$$u'\Omega^{-1}u = (\tilde{S}_{y,J} + \tilde{D}_{y,J} + \cdots + \tilde{D}_{y,1})'\Omega^{-1}(\tilde{S}_{y,J} + \tilde{D}_{y,J} + \cdots + \tilde{D}_{y,1})\ . \tag{3}$$

This equation can then be used for deriving the Whittle estimator. A linear model is now developed for evaluating the relationships among variables. First, we show the conventional form and then revise it as a multi-level model with varying time scales. A conventional linear model can be stated as:

$$y_t = x_t'\beta + u_t$$

where β and x_t are $k \times 1$ vectors, $\phi(L)u_t = \theta(L)a_t$, and a_t is the white noise. To simplify the calculation, we denote the covariance by $E(uu') = \Omega_u = \sigma^2\mathbf{\Sigma}_u$, where $u' = (u_1, u_2, ..., u_T)$. The related log likelihood function can then be constructed as:

$$\ln\mathbf{L} = -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln\sigma^2 - \frac{1}{2}\ln|\mathbf{\Sigma}_u| - \frac{1}{2\sigma^2}(y - x\beta)'\mathbf{\Sigma}_u^{-1}(y - x\beta)\ , \tag{4}$$

where y is a $T \times 1$ vector, x is a $T \times k$ matrix, and β is a $k \times 1$ vector. This is equivalent to

$$\begin{aligned}\ln\mathbf{L} = &-\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln\sigma^2 - \frac{1}{2}\ln|\mathbf{\Sigma}_u| \\ &- \frac{1}{2\sigma^2}(y - \tilde{S}_{x,J}\beta - \tilde{D}_{x,J}\beta - \cdots - \tilde{D}_{x,1}\beta)'\mathbf{\Sigma}_u^{-1}(y - \tilde{S}_{x,J}\beta - \tilde{D}_{x,J}\beta - \cdots - \tilde{D}_{x,1}\beta),\end{aligned}$$

Here $\tilde{S}_{x,j}$ and $\tilde{D}_{x,j}$ indicate the smooth and detailed parts, respectively, of a decomposition series at level j with scale 2^j, and each is a $T \times k$ matrix. An increase in the value of j is accompanied by a decrease in frequencies, suggesting the correlation is associated with a longer period of time. To examine the correlation at a given time scale, we construct a multi-level relationship model:

$$\ln \mathbf{L} = -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln\sigma^2 - \frac{1}{2}\ln|\mathbf{\Sigma}_u| - \frac{1}{2\sigma^2}(y - \tilde{S}_{x,J}\alpha - \tilde{D}_{x,J}\beta_J - \cdots - \tilde{D}_{x,1}\beta_1)'\mathbf{\Sigma}_u^{-1}(y - \tilde{S}_{x,J}\alpha - \tilde{D}_{x,J}\beta_J - \cdots - \tilde{D}_{x,1}\beta_1), \quad (5)$$

where α is the smooth-part relationship and β_j is the detailed-part relationship of level j. Again, for each increase in the level of j, the coefficients are associated with a correlation covering a longer length of time.

In equation (5), Σ_u^{-1} is unknown. To estimate this matrix, the Whittle method is used due to its simplicity and efficiency. Assuming u_t is a stationary process and $f_u(\lambda)$ is it's associated spectrum, we have

$$f_u(\lambda) = \frac{\sigma_u^2}{2\pi} g(\lambda;\Theta) \text{ and } g(\lambda;\Theta) = \left|\frac{1-\theta_1 e^{-i\lambda} - \ldots - \theta_q e^{-iq\lambda}}{1-\phi_1 e^{-i\lambda} - \ldots - \phi_p e^{-ip\lambda}}\right|^2.$$

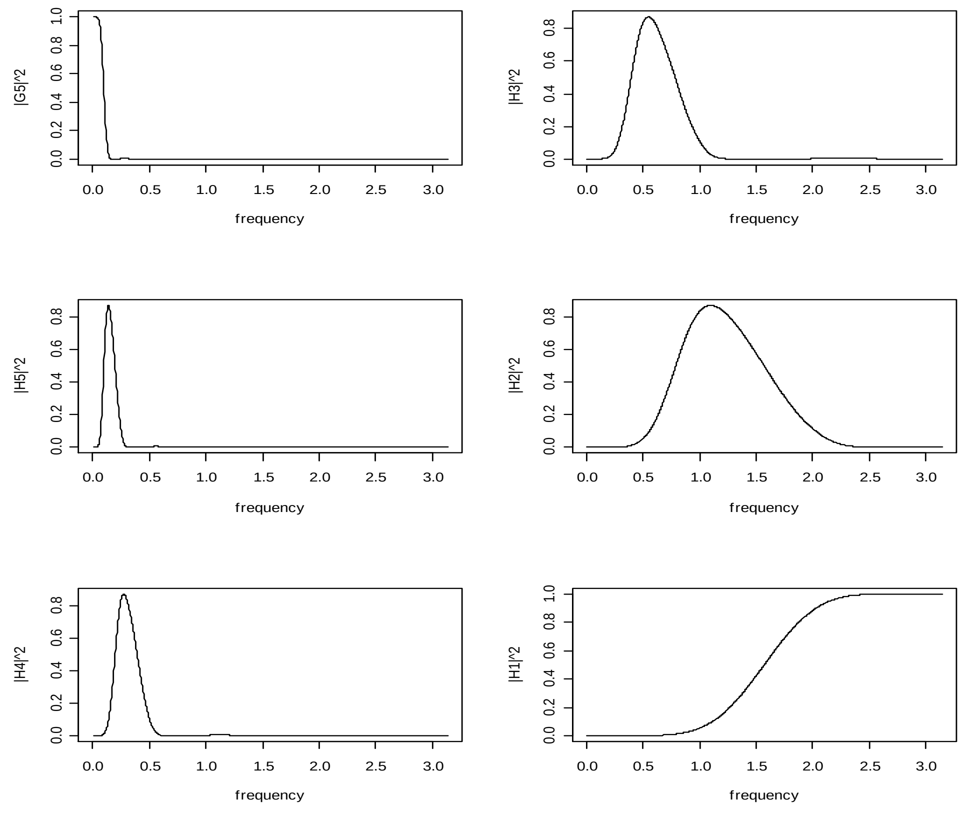
According to Beran (1994, (5.38) and (5.55)) and Priestley (1981, p. 741), we obtain $\Sigma_u^{-1} = \left[\delta(j-l)\right]_{j,l=1,\ldots,T}$, where $\hat{\delta}[k-l] = \frac{1}{T}\sum_{j=1}^{T}\frac{1}{g(\lambda_j;\Theta)} e^{i(k-l)\lambda_j}$, with harmonic frequencies $\lambda_j = 2\pi j/T$, $j = 1,\ldots, T^*$. $\frac{1}{T}\ln|\Sigma_u| = \frac{1}{2\pi}\int_{-\pi}^{\pi}\ln g(\lambda;\Theta)d\lambda = 0$. The log likelihood function can then be obtained as:

$$\ln L = -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln\sigma^2 - \frac{1}{\sigma^2}\sum_{k=1}^{[T/2]}\frac{1}{g(\lambda_k;\Theta)}\left|F_y(\lambda_k) - F_{\tilde{S}_{x,J}}(\lambda_k)\alpha - \sum_{j=1}^{J}F_{\tilde{D}_{x,j}}(\lambda_k)\beta_j\right|^2$$

where $F_z(\lambda_k) = \frac{1}{\sqrt{2\pi T}}\sum_{t=1}^{T} z_t e^{-i\lambda_k t}$ for any z_t. Thus, the number of the parameters associated with $\Phi = \{\sigma^2, \phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q, \alpha, \beta_1', \ldots, \beta_J'\}$ is equivalent to $M = p + q + J \times k + 3$. The Akaike's information criterion (AIC) and the Bayesian information criterion (BIC) can be applied to determine the appropriate level of decomposition.

Using the well-known Wald criterion, $\hat{\Phi}'R'V^{-1}R\hat{\Phi} \sim \chi_r^2$, where $V = R\mathbf{I}^{-1}R'$, we test a set of r linear restrictions on Φ, with the null hypothesis H_0: $R\Phi = 0$ against the alternative hypothesis H_1: $R\Phi \neq 0$. Given the appropriate Whittle estimator and wavelet scale, this test determines the significance of multi-level regression coefficients.

Figure 1: Spectra of Wavelet Coefficients at Various Levels



Note: The figures from top left to bottom right are spectra of wavelet coefficients $|G_5(\lambda)|^2$, $|H_5(\lambda)|^2$, $|H_4(\lambda)|^2$, $|H_3(\lambda)|^2$, $|H_2(\lambda)|^2$, and $|H_1(\lambda)|^2$, respectively. The subscript of $|H_j(\lambda)|^2$ corresponds to the spectrum of the wavelet coefficients at level j. The higher the level j is, the lower the frequencies are. The superimposition of these figures on each other offers a hint on the relationship among the spectra , and therefore on the Whittle estimator.

3. Simulations and Power Tests

Applying the well-known Daubechies wavelet, simulations and power tests are used to evaluate the asymptotic properties of estimators. Our Monte Carlo experiments involve series with different time lengths and models with various specifications. As previously pointed out, wavelet analysis is better equipped to process non-stationarity than differencing. For this reason, the explanatory variable used in our simulation is a non-stationary process. Following Equation (5), a time series can be decomposed as follows:

$$x_t = \tilde{S}_{x,Jt} + \tilde{D}_{x,Jt} + \cdots + \tilde{D}_{x,1t} .$$

The dependent variable can therefore be explained by:

$$y_t = \tilde{S}_{x,Jt}\alpha_J + \tilde{D}_{x,Jt}\beta_J + \cdots + \tilde{D}_{x,1t}\beta_1 + u_t . \tag{6}$$

Our multi-level model has an advantage over other models due to its ability to separate short term impacts on the dependent variable from that of long term. The impact of the smooth part associated with the highest level is reflected by α_J, while the more volatile impacts associated with the shorter-term detailed parts are measured by β_j. When j assumes a smaller value, the associated time span shortens.

Without losing generality, we analyze a disturbance term ε_t following an AR(1) process, and evaluate the test size as well as the power of each estimator. Let $x_t = \tilde{S}_{x,2t} + \tilde{D}_{x,2t} + \tilde{D}_{x,1t}$, $\varepsilon_t = 0.5\varepsilon_{t-1} + a_t$, a_t ~ NID(0,1) and the true values be $\alpha = 2$, $\beta_2 = 4$, $\beta_1 = 4$, $\phi = 0.5$, $\sigma^2 = 1$, and $J = 2$, respectively. The time lengths are set to vary from 100, 150 to 200 periods, each with 3000 replications.

Simulation results are shown in Table 1. The mean of estimated α of each simulated series is very close to the true value. Their corresponding standard errors are 0.4039, 0.3362, and 0.2789, respectively. As expected, the standard deviation decreases as the time length of a simulated series increases. Similar results are obtained for estimates of β_1, β_2, ϕ, and σ^2, with estimated means around their true values, and standard deviations vary inversely with sample size. These results suggest that estimators are consistent. Table 1 also reveals that when the time length increases from 100, to 150, and then to 200, the probability of rejecting the null hypothesis of $\alpha = 2.0$ varies from 6.97%, to 7.00%, and then to 5.37%, with significance level of 5%. As expected, for the null hypothesis $\alpha = 1.5$, the rejection probability increases to 26.13%, 33.93%, and 41.30%, respectively. Similar results are obtained for estimators of β_1, β_2, ϕ, and σ^2.

Table 1: Significance of Multi-level Regression Coefficients with Short Memory Disturbance

H_0: α =	0.00	0.50	1.00	1.50	2.0	2.50	3.00	3.50	4.00	Mean	Std. Error
T=100	0.9763	0.9277	0.685	0.2613	0.0697	0.252	0.6743	0.9307	0.9763	2.0056	0.4039
T=150	0.9783	0.970	0.8213	0.3393	0.070	0.34	0.824	0.9673	0.9777	2.0003	0.3362
T=200	0.9777	0.9747	0.908	0.4130	0.0537	0.405	0.907	0.9753	0.977	2.0024	0.2789
H_0: β_2 =	2.00	2.50	3.00	3.50	4.0	4.50	5.00	5.50	6.00	Mean	Std. Error
T=100	0.258	0.176	0.1137	0.078	0.066	0.0767	0.113	0.1767	0.2567	3.9819	1.9807
T=150	0.2467	0.173	0.1113	0.07	0.0617	0.0723	0.1027	0.159	0.2437	4.0034	1.9145
T=200	0.2517	0.1703	0.1083	0.076	0.0653	0.0847	0.116	0.1817	0.2703	3.9780	1.9793
H_0: β_1 =	2.00	2.50	3.00	3.50	4.0	4.50	5.00	5.50	6.00	Mean	Std. Error
T=100	0.457	0.3063	0.1807	0.0993	0.0727	0.0993	0.1783	0.304	0.4457	4.0292	1.3583
T=150	0.4577	0.3107	0.1847	0.0963	0.0727	0.0953	0.1747	0.302	0.4557	4.0077	1.3401
T=200	0.4783	0.324	0.1947	0.1137	0.084	0.095	0.17	0.3017	0.4397	4.0540	1.3571
H_0: ϕ =	-0.30	-1.00	0.10	0.30	0.50	0.70	0.90	1.10	1.30	Mean	Std. Error
T=100	0.542	0.3837	0.1257	0.0107	0.001	0.017	0.1853	0.462	0.576	0.4769	0.1070
T=150	0.6307	0.51	0.1963	0.011	0.000	0.0263	0.2907	0.5593	0.6457	0.4825	0.0735
T=200	0.721	0.622	0.3137	0.0173	0.000	0.03	0.4063	0.6407	0.7283	0.4866	0.0622
H_0: σ^2 =	0.60	0.70	0.80	0.90	1.0	1.10	1.20	1.30	1.40	Mean	Std. Error
T=100	0.8133	0.4343	0.1397	0.052	0.0973	0.2433	0.4677	0.6727	0.8327	0.9586	0.1399
T=150	0.9583	0.6983	0.2667	0.0607	0.0883	0.2677	0.536	0.7857	0.9237	0.9700	0.1152
T=200	0.9917	0.857	0.4123	0.0907	0.0773	0.278	0.6103	0.854	0.9613	0.9791	0.1005

Note: The size and power tests are based on 3000 replications, and the Daubechies recipe with filter length N = 6 is used to test parameter values in $y_t = \tilde{S}_{2t}\alpha + \tilde{D}_{2t}\beta_2 + \tilde{D}_{1t}\beta_1 + \varepsilon_t$ and $x_t = \tilde{S}_{2t} + \tilde{D}_{2t} + \tilde{D}_{1t}$, where $(1-0.5L)\varepsilon_t = a_t$, a_t ~ NID(0,1). True values of these parameters are α = 2, β_2 = 4, β_1 = 4, ϕ =0.5, and σ^2 =1. Time lengths are T=100, T=150, and T = 200. The significance level is at 5%, and x_t ~ I(1). The means of estimated α at different time lengths (100, 150, and 200) are 2.0056, 2.0003, and 2.0024, with standard deviations of 0.4039, 0.3362, and 0.2789, respectively. With the true value of α = 2.0, we obtain a consistent estimator for every sample size. Similar results have been obtained for β_2, β_1, ϕ and σ^2.

4. Empirical Analysis

We apply the stratified model to examine the stock return of Yulon Motor, a well-known auto manufacturer in Taiwan. The company, established in 1953, is listed at the Taiwan

Stock Exchange. Its market covers Taiwan, mainland China, and Hong Kong. Yulon is now one of the largest car makers in that region. The price series used in this study, covering a period between January 2, 2007 and December 30, 2011, is obtained from the Taiwan Stock Exchange. The capital market pricing model (CAPM) can be stated as:

$$R_{i,t} - \overline{R}_i = \beta_i (R_{m,t} - \overline{R}_m) + \varepsilon_t \,, \tag{7}$$

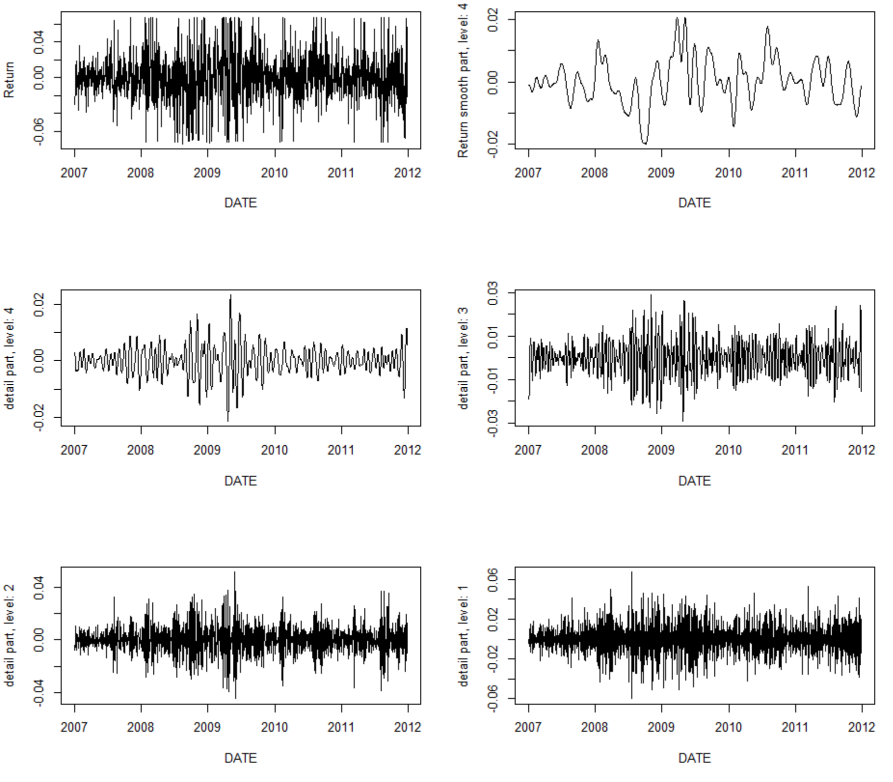
where $R_{i,t}$ is the rate of return on the ith security at time t, and $R_{m,t}$ the rate of return on the market portfolio. We use the Taiwan Stock Exchange Capital Weighted Index (TAIEX) as a proxy of prices of the market portfolio. Let ε_t be the stochastic disturbance. β_i denotes the beta coefficient of the ith security, a measure of the market (or systematic) risk of the security, which cannot be eliminated through diversification. It is well known that β_i measures the extent to which the ith security's rate of return moves with the market return. When β_i > 1, R_i is expected to be more volatile than R_m. The CAPM, shown in Equation (7), can be restated in terms of multi-level relationship:

$$R_{i,t} - \overline{R}_i = \tilde{S}_{m,Jt}\beta_{i,s} + \tilde{D}_{m,Jt}\beta_{i,J} + \cdots + \tilde{D}_{m,jt}\beta_{i,j} + \cdots + \tilde{D}_{m,1t}\beta_{i,1} + \varepsilon_t \,,$$

where $R_{m,t} - \overline{R}_m = \tilde{S}_{m,Jt} + \tilde{D}_{m,Jt} + \cdots + \tilde{D}_{m,jt} + \cdots + \tilde{D}_{m,1t}$, and $\beta_{i,j}$ indicates the beta coefficient of the ith security at level j.

We investigate the systematic risks based on different scales. A Daubechies wavelet with N = 6 is applied, and the highest level is J = 4. Applying wavelet analysis, the returns on Yulon stocks can be decomposed into several series, as shown in Figure 2. Each series reveals a specific interval of densities. The lower the level of j, the higher is the density. Using our analysis, investors can be grouped according to their trading strategies. Those who focus on long term returns, for example, are more interested in the fluctuations displayed by the top diagram of the right-hand side in Figure 2. On the other hand, those investors who focus on short-term fluctuations, would pay more attention to the pattern revealed at the bottom right. Thus, wavelet analysis allows us to examine the patterns of investment behavior even when the subject of investigation is the returns on a single stock.

Figure 2: Multi-level Decomposition of YULON Returns



Note: The left top figure shows the original daily returns of TSMC, from January 2, 2007 to December 30, 2011. The others reveal the decomposed returns at different levels. The higher is the level, the lower is the frequency.

Applying BIC, we can choose the best model for analyzing the relationship between Yolon returns and market returns. Table 2 points out that the model with MA(1) residuals is our choice.

Table 2: The Identification of ARMA Model Based on BIC

ARMA(p,q)	*q = 0*	*q = 1*	*q = 2*
$p = 0$	-7.22888	-7.73140	-7.72591
$p = 1$	-7.73139	-7.72572	-7.72083
$p = 2$	-7.72601	-7.72098	-7.71550

Note: BIC is used to determine the proper orders of ARMA for modeling YULON returns at $J = 4$. Based on the BIC criterion, MA(1) should be chosen. The Daubechies wavelet is with $N = 6$.

The results can be displayed as follows:

$$R_{i,t} - \bar{R}_i = \underset{(0.1646)}{1.4529}\,\tilde{S}_{m,4t} + \underset{(0.2292)}{1.3280}\,\tilde{D}_{m,4t} + \underset{(0.1407)}{1.1345}\,\tilde{D}_{m,3t} + \underset{(0.1113)}{1.4298}\,\tilde{D}_{m,2t} + \underset{(0.0631)}{1.0587}\,\tilde{D}_{m,1t} + \hat{\varepsilon}_t$$

where $\hat{\varepsilon}_t = \hat{a}_t + \underset{(0.0456)}{0.0177}\,\hat{a}_{t-1}$ and $\hat{\sigma}_a = 0.0205$.

Based on wavelet analysis, the frequency-based trading strategies are associated with the following scales or investment horizon: less than 6 trading days; 6 to 11 days; 12 to 23 days; 24 to 47 days; and over 48 days. The above equation displays the systematic risk associated with each of the strategies. For instance, those who trade securities with an investment horizon of less than six days, experience a systematic risk of 1.0587 with a standard error is 0.0631. Those who prefer an investment horizon between 6 and 11 trading days encounter a systematic risk of 1.4298 with a standard error is 0.1113. The corresponding *t* statistic for comparing these two risk measures is 2.90052 (= $\frac{1.4298 - 1.0587}{\sqrt{0.0631^2 + 0.1113^2}}$) at the significance level of 5%. Thus, our analysis reveals the systematic risk associating with each investment horizon or trading strategy.

5. Concluding Remarks

The proposition that economic agents are heterogeneous with bounded rationality has been confirmed by psychological experiments. Patterns of economic behavior of these agents are the subjects of study of heterogeneous agent models. The modeling efforts, however, focus basically on simulation.

In this paper, we go beyond simulation. Applying MODWT on time series, we are able to estimate multi-level correlations between returns on a stock and returns on the market portfolio. The error term has the flexibility to incorporate both long memory as well as short memory. Monte Carlo simulations suggest that our Whittle estimator is consistent and powerful. We find that systematic risk varies as the trading strategy changes its investment horizons. In addition, this risk does not change in proportion to the investment horizon.

The well-established capital asset pricing model is an analysis based on time domain. When the relationship between the returns on a single stock and the market return is examined in both time and frequency domain, the pricing of systematic risk varies according to investment horizon. Thus, our empirical results beg for an alternative investment theory to explain the stratified market phenomenon.

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